Physics 216/116 Winter 2019

Problem Set 1: Due 1/23

1) Complete the calculation of the induced mass of potential flow around a sphere, which was begun and discussed in class. In particular, show the energy of potential flow is

$$E = \rho\left[4\pi(\mathbf{A}\cdot\mathbf{u}) - V_0\frac{u^2}{2}\right] = m_{ik}\frac{u_i u_k}{2},$$

where $\mathbf{A}$ is the dipole moment of the flow and V_0 is the volume of the body in motion at $\mathbf{u}$. Compute m_{ik}, the induced mass tensor. What is its value for a sphere?

2) Consider a small body immersed in a fluid flow which oscillates. Derive the general relation between the velocity of the body and that of the fluid. What is the result for a spherical body of density ρ_0?

3) Derive the energy relation

$$\frac{\partial}{\partial t}\left(\rho\frac{v^2}{2} + \rho\epsilon\right) = -\nabla\cdot\left(\rho\mathbf{v}\left(\frac{v^2}{2} + \omega\right)\right)$$

from the continuity, Euler and energy equations. Here, ω is the enthalpy density.

4) a) Derive the dispersion relation for an azimuthally symmetric wave propagating along the $\hat{z}$ axis and in radius in an ideal incompressible, unbounded fluid rotating at $\mathbf{\Omega} = \Omega_0\hat{z}$.
   b) Now assume the fluid is bounded by a cylindrical wall at $r = R$. What is the profile of radial velocity?